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MODELLING THE MOTION OF A 2-ARM ROV

Marius R. Saure
Mechanical and Marine Engineering
Western Norway University of Applied Sciences
(HVL), Bergen, Norway
Sondre.h.i@hotmail.com

Sondre H. Iversen
Mechanical and Marine Engineering
Western Norway University of Applied Sciences
(HVL), Bergen, Norway
Marius.saure@gmail.com

Andreas B. Snekkevik
Mechanical and Marine Engineering
Western Norway University of Applied Sciences
(HVL), Bergen, Norway
Andreas_sne@hotmail.com

Rose Gebhardt
The Cooper Union
Mechanical Engineering
New York, NY, USA
chen30@cooper.edu

Zhiyang Chen
The Cooper Union
Mechanical Engineering
New York, NY, USA
gebhardt@cooper.edu

Christopher Mignano
The Cooper Union
Mechanical Engineering
New York, NY, USA
mignano@cooper.edu

Dirk Luchtenburg The
Cooper Union
Mechanical Engineering
New York, NY, USA
dluchten@cooper.edu

Thomas J. Impelluso
Mechanical and Marine Engineering
Western Norway University of Applied Sciences
(HVL), Bergen, Norway
tjm@hvl.no

ABSTRACT

Norway conducts operations on a variety of structures in the North Sea; e.g. oilrigs, monopole windmills, subsea trees. These structures often require subsea installation, observation, and maintenance. A remotely operated vehicle (ROV) can assist in these operations. Automation of intended motion is the desired goal. This paper researches the motion of an ROV induced by the motion of the robotic manipulators, motor torques, and added mass of fluid. This project builds upon a previous project that had one robotic arm; this time, there are two, but the method is unchanged. Furthermore, this work explores both the patterns in addressing such challenges, and an improved integration scheme. This research uses the Moving Frame Method (MFM) to carry out this project. In fact, this paper demonstrates the ease with which the MFM is extensible. Notable is that this work represents an international collaboration between an engineering school in Norway and one in the US. This work invites further research into improved numerical methods, solid/fluid interaction and the design of Autonomous Underwater Vehicles (AUV). AUVs beckon an era of Artificial Intelligence when machines think, communicate and learn. Rapidly deployable software implementations will be essential to this task.

NOMENCLATURE

$\{M\}$	Mass matrix
$\{M^*\}$	Reduced Mass matrix
$\{N^*\}$	Reduced non-linear velocity matrix
q	Generalized coordinates
$\dot{q}$	Generalized velocity
$\ddot{q}$	Generalized acceleration
R	Rotation matrix
$\mathbf{r}$	Absolute position vector
$\mathbf{s}$	Relative position vector
U	Potential energy
$\{\dot{X}\}$	List of velocities
δ	Variation
δW	Virtual work
$\delta\prod$	Variation of frame connection matrix
$\delta \overset{\bullet}{X}$	Variation of the generalized rates
δX	Virtual generalized displacement
$\delta\dot{X}$	Virtual generalized velocity
δq	Virtual essential generalized displacement

$\overleftrightarrow{\delta\pi}$ Virtual rotational (skew symmetric)
Ω Time rate of frame connection matrix
ω Angular velocity vector
$\overleftrightarrow{\omega}$ Skew symmetric angular velocity matrix
I_3, I_d 3×3 identity matrix
$J_C^{(\alpha)}$ 3×3 mass moment of inertia matrix
K Kinetic energy
L Lagrangian

INTRODUCTION

Engineering background

The Norwegian oil adventure began in 1969 with the discovery of one of the world's largest oil fields at sea, Ekofisk. This discovery precipitated a need for the development of new technology but within the fabric of Norwegian life such as safety and sustainability. Norwegians realized that remotely operated vehicles could overcome the challenges of hazardous environments.

The US Navy provided some of the first remotely operated vehicles (ROV) in the 1960's. In later years, the ROV has become an important tool within a number of fields such as the aquaculture and marine industry.

Today the ROVs are often custom made, making them especially applicable to specific tasks and situations. Some design elements are common, such as umbilical cables, multiple cameras, and one or two robotic arms [1]. Additionally, the frame contains thrusters to enable the ROV to move. However, vehicles designed for a specific task are often susceptible to rapid changes in their environment such as forces that alter that designed trajectory.

FIGURE 1. Typical ROV

Operating conditions expose the ROV to several different forces such as currency, buoyancy, and waves [2, 3, 4]. These forces affect the motion of the ROV, causing it to translate and rotate. Consequently, there is a significant increase in the risk of wear and damage on the ROV, and possible failure to achieve its goals. A pilot often manually operates the ROV to mitigate the risk of damage. The pilot continuously corrects the position and orientation of the vehicle. It is important to understand how the forces affect the motion of the ROV in order to correct the movements instantaneously. This is more effective than adjusting the ROV after the movement has already occurred. However, the incipience of smart machines and artificial intelligence beckons a more efficient and readily programmable means of predicting motion.

In this paper, an analysis of the forces and the resulting motions of the ROV will extend the work of a previous paper [5]. As important, we will introduce and deploy a new approach to dynamics: the Moving Frame Method.

The Moving Frame Method

This paper leverages its analysis of the new Moving Frame Method (MFM), as applied to multi-body systems.

Élie Cartan (1869-1951) [6] assigned a reference frame to each point of an object under study (a curve, a surface, Euclidean space itself). Then, using an orthonormal expansion, he expressed the rate of change of the frame in terms of the frame. The MFM leverages this by placing a reference frame on every moving link. However, then we need a *method to connect moving frames*. For this, we turn to Sophus Lie.

Marius Sophus Lie (1842-1899) developed the theory of continuous groups and their associated algebras. The MFM adopts the mathematics of rotation groups and their algebras, yet distils them to simple matrix multiplications. However, then we need a simplifying notation. For this, we turn to Frankel.

Ted Frankel [7] developed a compact notation in geometrical physics. The MFM adopts this notation to enable a methodology that is identical for both 2D and 3D analyses. The notation is also identical for single bodies and multi-body linked systems. In turn, this uplifts students' understanding from the conceptual to the pragmatic, enabling them to analyze machines of the 3D world. An introduction and pedagogical assessment may be found in the work of Impelluso [8]. Allow us to introduce the MFM.

The MFM has been used elsewhere. The MFM has been used to model how to stabilize ships at sea [9]. The MFM has been used to model Gyroscopic wave energy [10]. Finally, it has also been used to model a one-arm ROV [11].

This current study is new for several reasons. In this project, we demonstrate that the MFM and its notation, presents such simple commonalities, that the aforementioned domain areas are all modeled similarly. In this way, we continue the path to a common code. Furthermore, we demonstrate that a simple algebraic notation extends a 1-arm ROV model to a 2-arm model. In addition, we use a new numerical integration scheme. Finally, we show that the MFM makes international collaborations feasible, for this current work was conducted by a team of students in Bergen, Norway and New York City.

GENERAL INTRODUCTION TO MFM: SE(3)

A general multi-body system consists of multiple linked bodies. With α= 1, 2, 3, … as a superscripts for bodies, each individual body is endowed with its own moving Cartesian coordinate system:

$$s_c^{(\alpha)}(t) = \left\{ s_1^{(\alpha)}(t) \quad s_2^{(\alpha)}(t) \quad s_3^{(\alpha)}(t) \right\}^T \tag{1}$$

We define a body frame by partial derivatives of the coordinate functions:

$$\left\{ \mathbf{e}_1^{(\alpha)}(t) \quad \mathbf{e}_2^{(\alpha)}(t) \quad \mathbf{e}_3^{(\alpha)}(t) \right\} = \left\{ \partial / \partial s_1^{(\alpha)} \quad \partial / \partial s_2^{(\alpha)} \quad \partial / \partial s_3^{(\alpha)} \right\} \tag{2}$$

Thus, $\mathbf{e}^{(\alpha)}(t) = \left(\mathbf{e}_1^{(\alpha)}(t) \quad \mathbf{e}_2^{(\alpha)}(t) \quad \mathbf{e}_3^{(\alpha)}(t) \right)$ is a time-dependent moving frame, associated with the (α) moving body. Subscripts 1, 2, and 3 represent three orthogonal directions.

When necessary, we derive an inertial frame at the start of the analysis from the first body:

$$\left\{ \mathbf{e}_1^I \quad \mathbf{e}_2^I \quad \mathbf{e}_3^I \right\} = \left\{ \mathbf{e}_1^{(\alpha)}(0) \quad \mathbf{e}_2^{(\alpha)}(0) \quad \mathbf{e}_3^{(\alpha)}(0) \right\} \tag{3}$$

Kinematics of frames in general

Initially, we allow a body to translate. We designate the center of mass of the boat with subscript-C. We use the inertial frame when assessing the translation of the first body. We represent this first translation with “*x*,” reserving “*s*” for position vectors formulated in moving frames:

$$\mathbf{r}_c^{(\alpha)}(t) = \mathbf{e}^I x_c^{(\alpha)}(t) \tag{4}$$

In Equation (4) the frame is placed as a row vector before the components. With the use of this notation, the rotation matrices are viewed as matrix operators on columns of components [7].

The vector $s_c^{(\alpha+1/\alpha)}(t)$ represents the position to the center of mass $c^{(\alpha+1)}(t)$ a child body from the center of mass $c^{(\alpha)}(t)$ of the parent body. We express this vector in the parent frame as:

$$\mathbf{s}_c^{(\alpha+1/\alpha)}(t) = \mathbf{e}^{(\alpha)}(t) s_c^{(\alpha+1/\alpha)}(t) \tag{5}$$

Combined with the vector to locate the parent body from the inertial frame, the absolute location of the child frame is:

$$\mathbf{r}_c^{(\alpha+1)}(t) = \mathbf{r}_c^{(\alpha)}(t) + \mathbf{e}^{(\alpha)}(t) s_c^{(\alpha+1/\alpha)}(t) \tag{6}$$

To orient the moving frame, we use $R^{(\alpha)}(t)$, a 3×3 rotation matrix $R^{(\alpha)}(t)$. This notation expresses the rotation of body α vector-basis $\mathbf{e}^{(\alpha)}(t)$ from inertial vector-basis $\mathbf{e}^{(I)}$:

$$\mathbf{e}^{(\alpha)}(t) = \mathbf{e}^{(I)} R^{(\alpha)}(t) \tag{7}$$

The vector-basis $\mathbf{e}^{(\alpha+1)}(t)$ and the relative rotation of a body $(\alpha+1)$, is given by a relative rotation matrix $R^{(\alpha+1/\alpha)}(t)$ as:

$$\mathbf{e}^{(\alpha+1)}(t) = \mathbf{e}^{(\alpha)}(t) R^{(\alpha+1/\alpha)}(t) \tag{8}$$

This can also be expressed in the inertial frame by utilizing the group nature of SO(3):

$$\mathbf{e}^{(\alpha+1)}(t) = \mathbf{e}^{(I)} R^{(\alpha)}(t) R^{(\alpha+1/\alpha)}(t) = \mathbf{e}^{(I)} R^{(\alpha+1/\alpha)}(t) \tag{9}$$

The inverse of a rotation matrix is the transpose (a property of SO(3)):

$$\left(R^{(\alpha)}(t) \right)^{-1} = \left(R^{(\alpha)}(t) \right)^T \tag{10}$$

The time rate of frame rotation is:

$$\dot{\mathbf{e}}^{(\alpha)}(t) = \mathbf{e}^I \dot{R}^{(\alpha)}(t) \tag{11}$$

By post-multiplying both sides of equation (7) with $\left(R^{(\alpha)}(t) \right)^T$ (exploiting the orthogonality of SO(3)), and inserting the result in (11) we find:

$$\dot{\mathbf{e}}^{(\alpha)}(t) = \mathbf{e}^{(\alpha)}(t) \left(R^{(\alpha)}(t) \right)^T \dot{R}^{(\alpha)}(t) \tag{12}$$

We define the skew-symmetric angular velocity matrix. We note that this element is a member of the associated algebra, so(3):

$$\overleftrightarrow{\omega^{(\alpha)}(t)} = \left(R^{(\alpha)}(t) \right)^T \dot{R}^{(\alpha)}(t) = \begin{bmatrix} 0 & -\omega_3^{(\alpha)}(t) & \omega_2^{(\alpha)}(t) \\ \omega_3^{(\alpha)}(t) & 0 & -\omega_1^{(\alpha)}(t) \\ -\omega_2^{(\alpha)}(t) & \omega_1^{(\alpha)}(t) & 0 \end{bmatrix} \tag{13}$$

We can now write Equation (12) as:

$$\dot{\mathbf{e}}^{(\alpha)}(t) = \mathbf{e}^{(\alpha)}(t) \overleftrightarrow{\omega^{(\alpha)}(t)} \tag{14}$$

Thus, we have now obtained the time rate of frame rotation in its own frame. The skew-symmetric angular velocity matrix is isomorphic to angular velocity vector, when we associate the components with the moving frame:

$$\boldsymbol{\omega}^{(\alpha)}(t) = \mathbf{e}^{(\alpha)}(t) \begin{pmatrix} \omega_1^{(\alpha)}(t) \\ \omega_2^{(\alpha)}(t) \\ \omega_3^{(\alpha)}(t) \end{pmatrix} \tag{15}$$

The powerful distinction now, from the current pedagogy, is that the frame is time dependent.

Frame Connections Matrices

In this section, we group the rotation and displacement in one structure which we designate as a homogeneous transformation matrix. Denavit and Hartenberg [12] used such homogenous transformation matrices, but did not recognize at the time that such transformations were members of the Special Euclidean Group denoted as SE(3). The MFM recognizes and exploits the algebra, se(3), associated with the group SE(3). A more thorough development of the theory is found in reference [13].

The body-α *frame connection* is the combination of the frame and its location, expressed as:

$$\left(\mathbf{e}^{(\alpha)}(t) \quad \mathbf{r}^{(\alpha)}(t)\right) \tag{16}$$

The *inertial frame connection*, where bold **0** identifies the origin, is expressed as:

$$\left(\mathbf{e}^{I} \quad \mathbf{0}\right) \tag{17}$$

We define by a frame connection *matrix* $E^{(\alpha)}$ that accounts for both the rotation and translation:

$$E^{(\alpha)}(t) = \begin{bmatrix} R^{(\alpha)}(t) & x_c^{(\alpha)}(t) \\ 0_{1x3} & 1 \end{bmatrix} \tag{18}$$

As a result, we may state:

$$\left(\mathbf{e}^{(\alpha)}(t) \quad \mathbf{r}^{(\alpha)}(t)\right) = \left(\mathbf{e}^{(I)}(t) \quad 0\right) E^{(\alpha)}(t) \tag{19}$$

The notation recovers in (19) and (18) Equation (4) and (7).

The relation between the child body $(\alpha+1)$ frame and the parent body-α -frame is expressed by the *relative* connection matrix $E^{(\alpha+1/\alpha)}(t)$

$$\left(\mathbf{e}^{(\alpha+1)}(t) \quad \mathbf{r}^{(\alpha+1)}(t)\right) = \left(\mathbf{e}^{(\alpha)}(t) \quad \mathbf{r}^{(\alpha)}(t)\right) E^{(\alpha+1/\alpha)}(t) \tag{20}$$

Where:

$$E^{(\alpha+1/\alpha)}(t) = \begin{bmatrix} R^{(\alpha+1/\alpha)}(t) & s_c^{(\alpha+1/\alpha)}(t) \\ 0_{1x3} & 1 \end{bmatrix} \tag{21}$$

Equation (21) and (20) recovers (6) and (8).

2-ARM ROV ANALYSIS

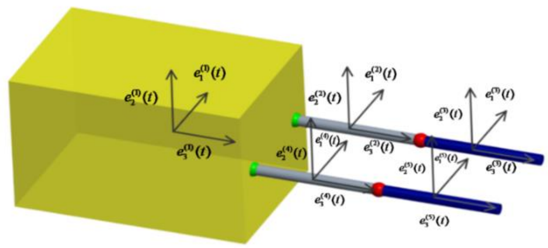

FIGURE 2. Model system description

The analysis commences with the first component in a linked structure: the frame of the ROV. From the ROV, we progress systematically to the first arm's proximal and distal link. Then, we return to the ROV and progress to the second arm. We assign a Cartesian coordinate frame to each component. We number the frames in ascending order, starting with the ROV and up to the second, distal, arm as the third frame. The following section contains a general overview of the MFM *as applied* to linked systems.
In Figure 2, the initial (2) axis is vertical; the initial (3) axis is directed along the links, and the (1) axis conforms to the right hand rule.

The frame connection matrix, $E^{(1)}(t)$, for the ROV includes the rotation matrix $R^{(1)}(t)$ and the position $x_c^{(1)}(t)$

$$E^{(1)}(t) = \begin{bmatrix} R^{(1)}(t) & x_c^{(1)}(t) \\ 0_{1x3} & 1 \end{bmatrix} \tag{22}$$

We obtain the time derivative of the frame connection matrix by taking the time derivative of each term:

$$\dot{E}^{(1)}(t) = \begin{bmatrix} \dot{R}^{(1)}(t) & \dot{x}_c^{(1)}(t) \\ 0_{1x3} & 0 \end{bmatrix} \tag{23}$$

As a member of SE(3), the inverse of the frame connection matrix is analytically known:

$$\left(E^{(1)}(t)\right)^{-1} = \begin{bmatrix} \left(R^{(1)}(t)\right)^{T} & -\left(R^{(1)}(t)\right)^{T} x_c^{(1)}(t) \\ 0_{1x3} & 1 \end{bmatrix} \tag{24}$$

We express the rate of change of the frame connection, in terms of the same frame connection. Thus, we progress:

$$\left(\dot{\mathbf{e}}^{(1)}(t) \quad \dot{\mathbf{r}}_c^{(1)}(t)\right) = \left(\mathbf{e}^{I} \quad 0\right) \dot{E}^{(1)}(t) \tag{25}$$

$$\left(\dot{\mathbf{e}}^{(1)}(t) \quad \dot{\mathbf{r}}_c^{(1)}(t)\right) = \left(\mathbf{e}^{(1)}(t) \quad \mathbf{r}_c^{(1)}(t)\right) \left(E^{(1)}(t)\right)^{-1} \dot{E}^{(1)}(t) \tag{26a}$$

We define the product of $\left(E^{(1)}(t)\right)^{-1}$ and $\dot{E}^{(1)}(t)$ the time rate of the frame connection matrix, as $\Omega^{(1)}(t)$:

$$\Omega^{(1)}(t) = \left(E^{1}(t)\right)^{-1} \dot{E}^{1}(t) \tag{26b}$$

Thus:

$$\left(\dot{\mathbf{e}}^{(1)}(t) \quad \dot{\mathbf{r}}_c^{(1)}(t)\right) = \left(\mathbf{e}^{(1)}(t) \quad \mathbf{r}_c^{(1)}(t)\right)\Omega^{(1)}(t) \tag{27}$$

When multiplying Eq. (26), we obtain the following:

$$\Omega^{(1)}(t) = \begin{bmatrix} \left(R^{(1)}(t)\right)^T \dot{R}^{(1)}(t) & \left(R^{(1)}(t)\right)^T \dot{x}_c^{(1)}(t) \\ 0_{1x3} & 0 \end{bmatrix} \tag{28}$$

$$\Omega^{(1)}(t) = \begin{bmatrix} \overleftrightarrow{\omega^{(1)}(t)} & v_c^{(1)}(t) \\ 0_{1x3} & 0 \end{bmatrix} \tag{29}$$

This matrix provides information about the linear and angular velocities of the coordinate frame attached to the ROV.

Thus, we can recover equations the following:

$$\overleftrightarrow{\omega^{(1)}(t)} = \left(R^{(1)}(t)\right)^T \dot{R}^{(1)}(t) \tag{30}$$

$$\dot{\mathbf{r}}_C^{(1)}(t) = \mathbf{e}^{(1)} v_C^{(1)}(t) = \mathbf{e}^{I} \dot{x}_C^{(1)}(t) \tag{31}$$

Kinematics of the second body: the proximal link, first arm

The first link of the robotic arm is the second body of the system. Attach a coordinate frame $\mathbf{e}^{(2)}(t)$ to the center of mass $C^{(1)}$ of the link:

$$\mathbf{e}^{(2)}(t) = \left(\mathbf{e}_1^{(2)}(t) \quad \mathbf{e}_2^{(2)}(t) \quad \mathbf{e}_3^{(2)}(t)\right) \tag{32}$$

We affirm the relative position from $\mathbf{e}^{(1)}(t)$ to $\mathbf{e}^{(2)}(t)$ by first translating from the center of mass $C^{(1)}$ of the ROV to the joint where the rotation happens. This translation from $C^{(1)}$ to the joint J_1 is obtained by moving in the 3-direction, half the total length of the ROV:

$$\mathbf{s}_j^{(1)} = \mathbf{e}^{(1)}(t) s_j^{(1)} = \mathbf{e}^{(1)}(t) \begin{pmatrix} h/3 \\ 0 \\ l^{(1)}/2 \end{pmatrix} \tag{33}$$

At the first joint, the rotation happens about the second axis (we are restricting this model to a revolute joint), which gives the following frame relation and rotation matrix about the shared 2-axis:

$$\mathbf{e}^{(2)}(t) = \mathbf{e}^{(1)}(t) R^{(2/1)}(t) = \mathbf{e}^{(1)}(t) \begin{bmatrix} \cos\varphi^{(1)}(t) & 0 & \sin\varphi^{(1)}(t) \\ 0 & 1 & 0 \\ -\sin\varphi^{(1)}(t) & 0 & \cos\varphi^{(1)}(t) \end{bmatrix} \tag{34}$$

Finally, we obtain the last translation from the joint J_1 to the center of mass of the first link by moving in the 3- direction, half the total length of the first link. We express this translation using the $\mathbf{e}^{(2)}$ -frame:

$$\mathbf{s}_c^{(2)}(t) = \mathbf{e}^{(2)}(t) s_c^{(2)} = \mathbf{e}^{(2)}(t) \begin{pmatrix} 0 \\ 0 \\ l^{(2)}/2 \end{pmatrix} \tag{35}$$

We express the structural relation between the first and second frame connections using the relative frame connection matrix $E^{(2/1)}(t)$:

$$\left(\mathbf{e}^{(2)}(t) \quad \mathbf{r}_c^{(2)}(t)\right) = \left(\mathbf{e}^{(1)}(t) \quad \mathbf{r}_c^{(1)}(t)\right) E^{(2/1)}(t) \tag{36}$$

We obtain this frame connection matrix by taking each of the steps described above: translating half the length of the ROV without rotation, rotating at the joint without translation, and finally translating half the length of the first link without rotation:

$$\begin{aligned} E^{(2/1)}(t) &= \begin{bmatrix} R^{(2/1)}(t) & s_c^{(2/1)}(t) \\ 0_{1x3} & 1 \end{bmatrix} \\ &= \begin{bmatrix} I & s_j^{(1)} \\ 0_{1x3} & 1 \end{bmatrix} \begin{bmatrix} R^{(2/1)}(t) & 0 \\ 0_{1x3} & 1 \end{bmatrix} \begin{bmatrix} I & s_c^{(2)} \\ 0_{1x3} & 1 \end{bmatrix} \end{aligned} \tag{37}$$

We carry out this matrix multiplication and obtain:

$$E^{(2/1)}(t) = \begin{bmatrix} R^{(2/1)}(t) & R^{(2/1)}(t) s_c^{(2)} + s_j^{(1)} \\ 0_3^T & 1 \end{bmatrix} \tag{38}$$

The second frame connection is related to the inertial frame connection through the absolute frame connection matrix $E^{(2)}(t)$:

$$\left(\mathbf{e}^{(2)}(t) \quad \mathbf{r}_c^{(2)}(t)\right) = \left(\mathbf{e}^{I} \quad 0\right) E^{(1)}(t) E^{(2/1)}(t) = \left(\mathbf{e}^{I} \quad 0\right) E^{(2)}(t) \tag{39}$$

$E^{(2)}(t)$ is of the form:

$$E^{(2)}(t) = \begin{bmatrix} R^{(2)}(t) & x_c^{(2)}(t) \\ 0_{1x3} & 1 \end{bmatrix} \tag{40}$$

When multiplying the above marices we obtain the following:

$$\begin{aligned} R^{(2)}(t) &= R^{(1)}(t) R^{(2/1)}(t) \\ x_c^{(2)}(t) &= R^{(1)}(t)\left(R^{(2/1)}(t) s_c^{(2)} + s_j^{(1)}\right) \end{aligned} \tag{41}$$

As with the ROV, the frame connection matrix, its inverse and derivative are used to calculate the time rate of the frame connection, $\Omega^{(2)}(t)$. From this, the rates are extracted. $\omega^{(2)}(t)$ are the components of the angular velocity matrix of the first link of the arm:

$$\omega^{(2)}(t) = \left(R^{(2/1)}(t)\right)^T \omega^{(1)}(t) + \omega^{(2/1)}(t) \tag{42}$$

We desired to simplify this. The rotation of the first link relative to the ROV body happens about one single axis, according to Eq. (34). Thus, the last term in (42) can be expressed by: $e_2\dot{\varphi}^{(1)}(t)$, where

$$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{43}$$

Thus:

$$\omega^{(2)}(t) = \left(R^{(2/1)}(t)\right)^T \omega^{(1)}(t) + e_2\dot{\varphi}^{(1)}(t) \tag{44}$$

And:

$$\begin{aligned} v_c^{(2)} &= \left(R^{(1)}(t)\left(\overleftrightarrow{R^{(2/1)}(t)s_c^{(2)}}\right)^T + R^{(1)}(t)\left(\overleftrightarrow{s_j^{(1)}}\right)^T \right)\omega^{(1)}(t) \\ &+ R^{(1)}(t)R^{(2/1)}(t)\left(\overleftrightarrow{s_c^{(2)}}\right)^T e_2\dot{\varphi}^{(1)}(t) \end{aligned} \tag{45}$$

Kinematics of the second body: the distal link, first arm

The second link of the arm is the third body of the system. Attach the next frame $\mathbf{e}^{(3)}(t)$, to the center of mass.

$$\mathbf{e}^{(3)}(t) = \left(\mathbf{e}_1^{(3)}(t) \quad \mathbf{e}_2^{(3)}(t) \quad \mathbf{e}_3^{(3)}(t)\right) \tag{46}$$

Again, we choose a simplification and assume a revolute joint enables the rotation about the shared 1st axis. Thus, the rotation matrix of the third frame from the second is a standard matrix:

$$R^{(3/2)}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta^{(2)}(t) & -\sin\theta^{(2)}(t) \\ 0 & \sin\theta^{(2)}(t) & \cos\theta^{(2)}(t) \end{bmatrix} \tag{47}$$

Continuing, we obtain the third frame from the second by translating the second half of the length of the first link (in the second frame), then rotating at the joint, and finally translating half the length of the second link to the center of mass $C^{(3)}$ (in the third frame) we designate these translations as:

$$\mathbf{s}_j^{(2)}(t) = \mathbf{e}^{(2)}(t)s_j^{(2)} = \mathbf{e}^{(2)}(t)\begin{pmatrix} 0 \\ 0 \\ l^{(2)}/2 \end{pmatrix} \tag{48a}$$

$$\mathbf{s}_c^{(3)}(t) = \mathbf{e}^{(3)}(t)s_c^{(3)} = \mathbf{e}^{(3)}(t)\begin{pmatrix} 0 \\ 0 \\ l^{(3)}/2 \end{pmatrix} \tag{48b}$$

The frame connection matrix that relates the two frames becomes:

$$\begin{aligned} E^{(3/2)}(t) &= \begin{bmatrix} I & s_j^{(2)} \\ 0_{1x3} & 1 \end{bmatrix}\begin{bmatrix} R^{(3/2)}(t) & 0 \\ 0_{1x3} & 1 \end{bmatrix}\begin{bmatrix} I & s_c^{(3)} \\ 0_{1x3} & 1 \end{bmatrix} \\ &= \begin{bmatrix} R^{(3/2)}(t) & R^{(3/2)}(t)s_c^{(3)} + s_j^{(2)} \\ 0_{1x3} & 1 \end{bmatrix} \end{aligned} \tag{49}$$

We can now establish the relative frame connection matrix $E^{(3/2)}(t)$:

$$\left(\mathbf{e}^{(3)}(t) \quad \mathbf{r}_c^{(3)}(t)\right) = \left(\mathbf{e}^{(2)}(t) \quad \mathbf{r}_c^{(2)}(t)\right)E^{(3/2)}(t) \tag{50}$$

We can then establish the absolute frame connection matrix $E^{(3)}(t)$

$$\begin{aligned} \left(\mathbf{e}^{(3)}(t) \quad \mathbf{r}_c^{(3)}(t)\right) &= \left(\mathbf{e}^{I} \quad 0\right)E^{(1)}(t)E^{(2/1)}(t)E^{(3/2)}(t) \\ &= \left(\mathbf{e}^{I} \quad 0\right)E^{(3)}(t) \end{aligned} \tag{51}$$

$E^{(3)}(t)$ is the absolute frame connection matrix of the third frame from the inertial and is on the form:

$$E^{(3)}(t) = \begin{bmatrix} R^{(3)}(t) & x_c^{(3)}(t) \\ 0_{1x3} & 1 \end{bmatrix} \tag{52}$$

By multiplying the matrices in the above equation, one can obtain the absolute rotation $R^{(3)}(t)$ and position $x_c^{(3)}(t)$ of the third frame from the inertia:

$$R^{(3)}(t) = R^{(1)}(t)R^{(2/1)}(t)R^{(3/2)}(t) \tag{53}$$

$$\begin{aligned} x_c^{(3)}(t) &= R^{(1)}(t)R^{(2/1)}(t)\left(R^{(3/2)}(t)s_c^{(3)} + s_j^{(2)}\right) \\ &+ \left(R^{(1)}(t)R^{(2/1)}(t)s_c^{(2)} + R^{(1)}(t)s_j^{(1)}\right) \end{aligned} \tag{54}$$

Through further manipulations similar to the previous links, we obtain the rates for the second link. The angular velocity vector of the third frame is:

$$\begin{aligned} \omega^{(3)}(t) &= \left(R^{(3/2)}(t)\right)^T \left(R^{(2/1)}(t)\right)^T \omega^{(1)}(t) \\ &+ \left(R^{(3/2)}(t)\right)^T e_2\dot{\varphi}^{(1)}(t) + e_1\dot{\varphi}^{(2)}(t) \end{aligned} \tag{55}$$

The linear velocity vector of the third frame from the inertial:

$$
\begin{aligned}
v^{(3)}(t) = &\\
&\begin{pmatrix}
R^{(1)}(t)\left(\overleftrightarrow{R^{(2/1)}(t)R^{(3/2)}(t)s_c^{(3)}}\right)^T + \\
R^{(1)}(t)\left(\overleftrightarrow{R^{(2/1)}(t)s_j^{(2)}}\right)^T + \\
R^{(1)}(t)\left(\overleftrightarrow{R^{(2/1)}(t)s_c^{(2)}}\right)^T + \\
R^{(1)}(t)\left(\overleftrightarrow{s_j^{(1)}}\right)^T
\end{pmatrix}\omega^{(1)}(t) + \\
&\begin{pmatrix}
R^{(1)}(t)R^{(2/1)}(t)\left(\overleftrightarrow{R^{(3/2)}(t)s_c^{(3)}}\right)^T + \\
R^{(1)}(t)R^{(2/1)}(t)\left(\overleftrightarrow{s_j^{(2)}}\right)^T + \\
R^{(1)}(t)R^{(2/1)}(t)\left(\overleftrightarrow{s_c^{(2)}}\right)^T
\end{pmatrix}\omega^{(2/1)}(t) + \\
&R^{(1)}(t)R^{(2/1)}(t)R^{(3/2)}(t)\left(\overleftrightarrow{s_c^{(3)}}\right)^T \omega^{(3/2)}(t)
\end{aligned}
\tag{56}
$$

We still desired to simplify this. The rotation of the both links relative to the ROV body happens about one single axis each, and we use this:

$$
\begin{aligned}
\omega^{(3/2)}(t) &= e_1\dot{\varphi}^{(2)}(t) \\
\omega^{(2/1)}(t) &= e_2\dot{\varphi}^{(1)}(t)
\end{aligned}
\tag{57}
$$

Kinematics of the fourth body: proximal and distal arm

The entire second arm has the same notation as the first. Only difference will be the translation between the frames from center of mas of the boat, to the first link:

$$
\mathbf{s}_j^{(3)}(t) = \mathbf{e}^{(1)}(t)\, s_j^{(3)} = \mathbf{e}^{(1)}(t)\begin{pmatrix} -h/3 \\ 0 \\ l^{(1)}/2 \end{pmatrix}
\tag{58}
$$

The other difference will be the notational designation of the angles, wich will be notated as $\psi^{(1)}(t)$ and $\psi^{(2)}(t)$. This give us the following omegas and xdot for the first link:

$$
\omega^{(4)} = \left(R^{(4/1)}(t)\right)^T \omega^{(1)}(t) + e_2\dot{\psi}^{(1)}(t)
\tag{59}
$$

$$
\begin{aligned}
v_c^{(4)} = &\left(R^{(1)}(t)\left(\overleftrightarrow{R^{(4/1)}(t)s_c^{(4)}}\right)^T + R^{(1)}(t)\left(\overleftrightarrow{s_j^{(3)}}\right)^T \right)\omega^{(1)}(t) \\
&+ R^{(1)}(t)R^{(4/1)}(t)\left(\overleftrightarrow{s_c^{(4)}}\right)^T e_2\dot{\psi}^{(1)}(t)
\end{aligned}
\tag{60}
$$

Then, the distal link:

$$
\begin{aligned}
\omega^{(5)}(t) = &\left(R^{(5/4)}(t)\right)^T \left(R^{(4/1)}(t)\right)^T \omega^{(1)}(t) \\
&+ \left(R^{(5/4)}(t)\right)^T e_2\dot{\psi}^{(1)}(t) + e_1\dot{\psi}^{(2)}(t)
\end{aligned}
\tag{61}
$$

$$
\begin{aligned}
v^{(5)}(t) = &\\
&\begin{pmatrix}
R^{(1)}(t)\left(\overleftrightarrow{R^{(4/1)}(t)R^{(5/4)}(t)s_c^{(5)}}\right)^T + \\
R^{(1)}(t)\left(\overleftrightarrow{R^{(4/1)}(t)s_j^{(4)}}\right)^T + \\
R^{(1)}(t)\left(\overleftrightarrow{R^{(4/1)}(t)s_c^{(4)}}\right)^T + \\
R^{(1)}(t)\left(\overleftrightarrow{s_j^{(3)}}\right)^T
\end{pmatrix}\omega^{(1)}(t) + \\
&\begin{pmatrix}
R^{(1)}(t)R^{(4/1)}(t)\left(\overleftrightarrow{R^{(5/4)}(t)s_c^{(5)}}\right)^T + \\
R^{(1)}(t)R^{(4/1)}(t)\left(\overleftrightarrow{s_j^{(4)}}\right)^T + \\
R^{(1)}(t)R^{(4/1)}(t)\left(\overleftrightarrow{s_c^{(4)}}\right)^T
\end{pmatrix}e_2\dot{\psi}^{(1)}(t) + \\
&R^{(1)}(t)R^{(4/1)}(t)R^{(5/4)}(t)\left(\overleftrightarrow{s_c^{(5)}}\right)^T e_1\dot{\psi}^{(2)}(t)
\end{aligned}
\tag{62}
$$

GENERALIZED COORDINATES

The first goal is to exploit a minimal set of generalized coordinates. To do this, we must find a more efficient means to express Eqs. (30), (31) (44), (45), (55), (56), (59), (60), (61) and (62). We have so far established generalized Cartesian velocities stated in a 27x1 column matrix $\{\dot{X}(t)\}$ in (X) (wherein, each individual term represents three components).

$$
\{\dot{X}(t)\} \equiv \begin{pmatrix} \omega^{(1)}(t) \\ \dot{x}_C^{(2)}(t) \\ \omega^{(2)}(t) \\ \dot{x}_C^{(3)}(t) \\ \omega^{(3)}(t) \\ \dot{x}_C^{(4)}(t) \\ \omega^{(4)}(t) \\ \dot{x}_C^{(5)}(t) \\ \omega^{(5)}(t) \end{pmatrix}
\qquad
\{\dot{q}(t)\} = \begin{pmatrix} \omega^{(1)}(t) \\ \dot{\varphi}^{(1)}(t) \\ \dot{\varphi}^{(2)}(t) \\ \dot{\psi}^{(1)}(t) \\ \dot{\psi}^{(2)}(t) \end{pmatrix}
\tag{63}
$$

However, these can be expressed by the translational velocity and angular velocity $\omega^{(1)}(t)$ of the ROV,the angular speed of the gimbal $\dot{\theta}(t)$ and angular speed of the disk $\dot{\phi}(t)$. We refer to these as the essential generalized velocities. They are represented in a 5x1 column matrix $\{\dot{q}(t)\}$ as shown in Eq. (63). By means of Eqs. (30), (31) (44), (45), (55), (56), (59), (60), (61) and (62).we relate the generalized velocities linearly with the essential generalized velocities using:

$$\left\{\dot{X}(t)\right\} = \left[B(t)\right]\left\{\dot{q}(t)\right\} \tag{64a}$$

$\left[B(t)\right]$ represents a 27x7 matrix whose non-zero entries are:

$$\begin{bmatrix} I & \underset{3\times1}{0} & \underset{3\times1}{0} & \underset{3\times1}{0} & \underset{3\times1}{0} \\ B_{21} & R^{(1)}(t)R^{(2/1)}(t)\left(\overleftrightarrow{s_c^{(2)}}\right)^T e_2 & \underset{3\times1}{0} & \underset{3\times1}{0} & \underset{3\times1}{0} \\ B_{31} & e_2 & \underset{3\times1}{0} & \underset{3\times1}{0} & \underset{3\times1}{0} \\ B_{41} & B_{42} & R^{(1)}(t)R^{(2/1)}(t)R^{(3/2)}(t)\left(\overleftrightarrow{s_c^{(3)}}\right)^T e_1 & \underset{3\times1}{0} & \underset{3\times1}{0} \\ B_{51} & \left(R^{(3/2)}(t)\right)^T e_2 & e_1 & \underset{3\times1}{0} & \underset{3\times1}{0} \\ B_{61} & \underset{3\times1}{0} & \underset{3\times1}{0} & R^{(1)}(t)R^{(4/1)}(t)\left(\overleftrightarrow{s_c^{(4)}}\right)^T e_2 & \underset{3\times1}{0} \\ B_{71} & \underset{3\times1}{0} & \underset{3\times1}{0} & e_2 & \underset{3\times1}{0} \\ B_{81} & \underset{3\times1}{0} & \underset{3\times1}{0} & B_{84} & B_{85} \\ B_{91} & \underset{3\times1}{0} & \underset{3\times1}{0} & \left(R^{(5/4)}(t)\right)^T e_2 & e_1 \end{bmatrix} \tag{64b}$$

Block terms are defined in the appendix

We now set aside this B matrix and turn to kinetics. Until otherwise noted, we will return to considering the generalized Cartesian velocities, $\left\{\dot{X}(t)\right\}$, and hold the essential generalized velocities, $\left\{\dot{q}(t)\right\}$, in abeyance until they are needed.

KINETICS

Kinetic energy K includes translation energy and rotational energy regarding each center of mass.

$$K^{(\alpha)} = \frac{1}{2}\left\{\left(\dot{x}_C^{(\alpha)}\right)^T m^{(\alpha)}\dot{x}_C^{(\alpha)} + \left(\omega^{(\alpha)}\right)^T J_C^{(\alpha)}\omega^{(\alpha)}\right\} \tag{65}$$

We use this in Hamilton's Principle, reformed as the Principle of Virtual work with all work (conservative and non-conservative) on the right side and where the Lagrangian (and its variation) now consists only of the kinetic energy of translation and rotation.

$$\delta\int_{t_0}^{t_1} K^{(\alpha)}(t)\,dt = -\delta\int_{t_0}^{t_1} W^{(\alpha)}(t)\,dt \tag{66}$$

Variations

We will need to take derivatives in the "direction" of the variation. The directional derivatives with ε are called the Gâteaux-derivatives in the functional space theory. The position vector $\mathbf{r}_c^{(\alpha)}$ is expressed as $\mathbf{r}_c^{(\alpha)} = \mathbf{e}^I x_c^{(\alpha)}$.This enables us to express the variation of the translation of a body-α from an inertial frame. Noting that $\delta\mathbf{e}^I = 0$, we find:

$$\delta r_C^{(\alpha)} = e^I \delta x_C^{(\alpha)} \tag{67}$$

The commutativity of mixed partials readily holds for translational velocity and one obtains the obvious:

$$\delta\dot{x}_C^{(\alpha)}(t) = \left(\frac{d}{dt}\delta x_C^{(\alpha)}(t)\right) \tag{68}$$

However, the variation of the angular velocity is restricted in 3D space. This was found by Murakami[10] and independently by Holm [14]. First, we define the following term:

$$\overleftrightarrow{\delta\pi^{(\alpha)}(t)} = \left(R^{(\alpha)}(t)\right)^T \delta R^{(\alpha)}(t) \tag{69}$$

Equation (69) term does not exist in is unvaried form. It defines the virtual frame-rotation vector $\delta\pi^{(\alpha)}$, in the same way as the angular velocity matrix defined the angular velocity in Eq. (13):

$$\delta\pi^{(\alpha)}(t) = \mathbf{e}^{(\alpha)}(t)\delta\pi^{(\alpha)}(t) \tag{70}$$

By ensuring the commutativity of mixed partials (time and variation with regard to the directional derivative of the variation parameter), we arrive at a restriction. We find that the variation of the angular velocity depends on the virtual frame rotation, referred to as restricted variation of virtual angular velocity:

$$\delta\omega^{(\alpha)} = \delta\dot{\pi} + \overleftrightarrow{\omega^{(\alpha)}}\delta\pi^{(\alpha)} \tag{71}$$

Moment versus virtual rotation represent a natural pair. They are conjugate to the moment M expressed with the body frame. Moment versus virtual rotation is a natural pair: Hamilton's principle, which yields Euler's equation. This was the weakest point in the classical multibody dynamics. Wittenburg [15, 16] postulated the principle of virtual power to use the weighted form of Euler's equation by the virtual angular velocity. However, moment and omega define the power, not the work. This was the weakest point in the classical multibody dynamics and the MFM has now rectified this. To take the variation we collate the unrestricted virtual generalized displacements $\left\{\delta X\right\}$:

$$\left\{\delta X\right\} = \begin{pmatrix} \delta x^{(1)} \\ \delta\pi^{(1)} \\ \delta x^{(2)} \\ \delta\pi^{(2)} \\ \delta x^{(3)} \\ \delta\pi^{(3)} \\ \delta x^{(4)} \\ \delta\pi^{(4)} \\ \delta x^{(5)} \\ \delta\pi^{(5)} \end{pmatrix} \tag{72}$$

The next step is to structure the relationships in Eq. (68) and (70). To accomplish this, we first define a system which consists of block matrices on the diagonal, and zero elsewhere.

The diagonals contain the skew symmetric angular velocity matrix of each body, with zero elsewhere as follows

$$[D]_{diagonal} = \begin{bmatrix} 0_{3\times3} & \overleftrightarrow{\omega^{(1)}(t)} & 0_{3\times3} & \overleftrightarrow{\omega^{(i)}(t)} & \dots & 0_{3\times3} & \overleftrightarrow{\omega^{(5)}(t)} \end{bmatrix} \quad (73)$$

$[D]$ is skew symmetric. Thus: $[D]^T = -[D]$. Continuing, we combine (X) and (X) into one equation is as follows:

$$\{\delta \dot{X}\} = \left\{\delta \overset{\bullet}{X}\right\} + [D]\{\delta X\} \quad (74)$$

Finally, there is one important issue. The system generalized velocity $\{\dot{X}(t)\}$ and the essential generalized velocity $\{\dot{q}(t)\}$ are linearly related through the $[B(t)]$-matrix (X). In the same way, since they represent derivatives, the same B matrix relates the virtual generalized displacements and the essential generalized coordinates.

$$\{\delta X(t)\} = [B(t)]\{\delta q(t)\} \quad (75)$$

Principle of virtual work

Hamilton's Principle is for a system with conservative force. Engineers, however, developed the Principle of Virtual Work that includes non-conservative forces. However, to use it, we ignore the potential energy and absorb all applied forces into the Work done on the system. We obtain the variation of the Lagrangian:

$$\delta \hat{L} = \{\delta \dot{X}(t)\}^T [M]\{\dot{X}(t)\} \quad (76)$$

We obtain the variation of the work, δW , done by $\{F(t)\}$:

$$\delta W = \{\delta X\}^T \{F(t)\} \quad (77)$$

For the applied forces, we have:

$$\{F(t)\} = \begin{pmatrix} F_C^{(1)I}(t) \\ M_C^{(1)}(t) \\ F_C^{(2)I}(t) \\ M_C^{(2)}(t) \\ F_C^{(3)I}(t) \\ M_C^{(3)}(t) \\ F_C^{(4)I}(t) \\ M_C^{(4)}(t) \\ F_C^{(5)I}(t) \\ M_C^{(5)}(t) \end{pmatrix} = \begin{pmatrix} F_b^I - m^{(1)} g e_2 \\ -M_m^{(1)}(t) e_2 \\ -m^{(2)} g e_2 \\ M_m^{(1)}(t) e_2 - M_m^{(2)}(t) e_1 \\ -m^{(3)} g e_2 \\ M_m^{(2)}(t) e_1 \\ -m^{(4)} g e_4 \\ M_m^{4}(t) e_5 - M_m^{(5)}(t) e_4 \\ -m^{(5)} g e_4 \\ M_m^{(5)}(t) e_4 \end{pmatrix} \quad (78)$$

We interpret these forces as follows:

$F_b^I =$	Force from buoyancy
$m^{(1)} g e_2 =$	Gravitational force on ROV main body
$-M_m^{(1)}(t) e_2 =$	Reverse moment on ROV caused by the first motor
$M_m^{(1)}(t) e_2 =$	Forward moment on the proximal arm by the first motor
$-m^{(2)} g e_2 =$	Gravitational force on the first link of the arm
$-M_m^{(2)}(t) e_1 =$	Reverse moment on distal arm caused by second motor
$M_m^{(2)}(t) e_1 =$	Forward moment on distal arm caused by second motor
$-m^{(3)} g e_2 =$	Gravitational force on the second link
$-m^{(4)} g e_4 =$	Gravitational force on the third link
$-M_m^{(5)}(t) e_4 =$	Reverse moment on proximal arm by the third motor
$M_m^{4}(t) e_5 =$	Forward moment on proximal arm by the third motor
$-m^{(5)} g e_4 =$	Gravitational force on the fourth link
$M_m^{(5)}(t) e_4 =$	Forward moment on distal arm by the fourth motor

The final form of the variation of the action becomes:

$$\int_{t_0}^{t_1} \left(\{\delta \dot{X}(t)\}^T [M]\{\dot{X}(t)\} + \{\delta X(t)\}^T \{F(t)\} \right) dt = 0 \quad (79)$$

Equation of motion

By making all the substitutions and carrying out the calculus of variations, one obtains the following (where steps are skipped and only critical plateaus provided):

Define the following:

$$[M^*(t)] \equiv [B(t)]^T [M][B(t)] \quad (80)$$

$$[N^*(t)] \equiv [B(t)]^T \left([M(t)][\dot{B}(t)] + [D(t)][M][B(t)] \right) \quad (81)$$

$$\{F^*(t)\} = [B(t)]^T \{F(t)\} \quad (82)$$

$$[M^*(t)]\{\ddot{q}(t)\} + [N^*(t)]\{\dot{q}(t)\} = \{F^*(t)\} \quad (83)$$

The result is a series of five coupled homogenous differential equations.

Updating the ROV's rotation matrix

The rotation matrices for the two arms are standard, due to the derivation from revolute joints. However, we must know the rotation matrix of the ROV for several reasons. First, it is required I the updating of the B matrix. Second, it is required to apply added mass forces. Finally, we need it for visualization.

We must reconstruct the rotation matrix of the ROV from the angular velocity. We must compute the rotation matrix $R^{(1)}(t)$ by solving the following equation:

$$\dot{R}^{(1)}(t) = R^{(1)}\overleftrightarrow{\omega^{(1)}(t)} \tag{84}$$

Let us assume for a moment that $\overleftrightarrow{\omega^{(1)}(t)}$ is constant and is designated as $\vec{\omega}_0$. Then, with initial value $R(0)$, the solution is:

$$R^{(1)}(t) = R(0)\exp(t\overleftrightarrow{\omega_0}) \tag{85}$$

There does exist a known analytical, closed form solution to Eq. (84), but only for cases in which $\vec{\omega}_0$ is constant. It derives from the Cayley Hamilton Theorem and is known as the Rodriguez' rotation formula to obtain a series expansion of the exponential of a matrix.

$$R(t+\Delta t) = R(t)\left(\begin{array}{l} I_d + \dfrac{\overleftrightarrow{\omega_0(t+\Delta t)}}{\|\omega(t+\Delta t)\|}\sin(t\|\omega_0(t+\Delta t)\|) \\ \left(\dfrac{\overleftrightarrow{\omega_0(t+\Delta t)}}{\|\omega(t+\Delta t)\|}\right)^2 (1-\cos(t\|\omega_0(t+\Delta t)\|)) \end{array} \right) \tag{86}$$

The difficulty is that we do not have a constant angular velocity matrix. However, we can approximate its constancy in each time step of the numerical integration.

In principle, one need only average this over two-time steps using a central difference approximation. However, for ease of first pass coding, we violate this rule by assuming a constant value at the start of each time step:

$$\omega(t+\Delta t/2) \equiv (\omega(t) + \omega(t+\Delta t/2))/2 \tag{87}$$

COLLABORATION

This paper is the result of a collaborative research effort between a group of students from Western Norway University of Applied Sciences in Bergen, Norway and another group of students from the Cooper Union in New York, USA.

Approach

Both groups of students are locally mentored, but they share their questions, knowledge and insights via email and skype. The goal has been to independently extract the equations of motion for a double-armed ROV.

Implementation

The Norway team implements the integration of the equations of motion and visualization of the ROV motion in JavaScripts and WebGL [17]. The Cooper team focuses on implementation in Python. While the Norway team is engaged in a similar project for a crane on a ship and is implementing an automated approach to the B assembly, the team in NYC has independently arrived at the same point. Clearly, this new method has opened vistas for advanced dynamics work and international collaboration at the undergraduate level

The MFM is new. Companies or Universities may wonder why the need to switch methods. We assert that this project was conducted by undergraduate students. The MVM enables advanced work in dynamics while encouraging collaboration.

RESULTS AND DISCUSSION

The goals of this project were at the intersection of research and education. This project represented an ambitious attempt to do three things at once: 1) initiate a collaboration between a school in NYC and in Norway, 2) conduct a first international senior design project with diverse students, and 3) advance ongoing research into ROV dynamics. We did not completely succeed. We affirm that students at both schools were able to secure the equations of motion of the ROV system, under applied driving torques on a dual arm system; and this was the most essential aspect of the research facet. The MFM is new, yet consistent. Last year's simpler project secured the equations and develop the 3D code for one arm, conducted at one school in a longer time frame. The reader may view last year's site:

http://home.hib.no/prosjekter/dynamics/2019/ROV

This year, we are not able to confirm coding success. Despite an ongoing bug in both visualization codes (one, which we expect to be found, mostly likely within a few weeks—as is often the case—and, if so, we will report that web site), both teams reported positive impressions from this collaborative project. After all, the primary focus of this work was education, collaboration and dissemination of a new method.

The MFM is daunting on first appearance. A casual reader immediately feels secure in the notation and methodology: one in which the notation for 2D and 3D, single bodies and multi-bodies is identical; however, that impression belies potential coding complexities. All members of this project—advisors and students—bear some responsibility for not anticipating the coding complexities.

The project was successful in that the students at Cooper Union took one approach after working out the critical B matrix. They implemented a process to automate the construction of the B matrix. They coded the results in Python.

The students at HVL took a second approach, after securing the B matrix. They coded the results in Javascript.

The kinematic analysis of the ROV that takes into account the degrees of freedom of all involved bodies and leads to the B-matrix (Eq. 64a) is quite long. In general, the number of bodies in the kinematic chain determines the complexity of the kinematic problem. The expressions in the B-matrix might be elaborate but are systematic. The kinematic relations for angular and linear speed have the same generic form for a particular joint connecting two bodies. The ROV, considered in this paper, only has revolute joints. The procedure for obtaining the B-matrix starts at the main body and moves `downstream` the kinematic chain to obtain the velocities of body one to four. The B-matrix (and Bdot matrix) are obtained by simple recursive relationships. Thus, the process of finding the B-matrix can be easily automated by specifying the dimensions of all bodies that constitute the ROV, the joint connections between the bodies, specification of the main body, and the direction of the kinematic chain.

The Cooper students report that the collaboration has been an invaluable addition to their education. In particular, the Norwegian students, being older and farther along in their education, were able to convey new insights into the MFM methodology. In addition, the Cooper students felt that the Norwegian students seemed to have a more practical real-world understanding of the MFM and dynamics. The students, however, rose above their concerns. It must be stated that the Norwegian students were working under the guidance of one of the developers of the MFM. As a result of these experiences, the Cooper students have been inspired to travel and study abroad. All the students report having been inspired by this project and the collaboration—and this is our definitive success.

The MFM allows for a rapid, extensible analysis of multi-body systems. Its first power is that the notation for rigid multi-body dynamics, relying on SE(3)/se(3) is extremely similar for the notation for rigid sing-body dynamics, relying on SO(3)/so(3); and, even then, the notation for 3D and 2D analyses are the same. Such simple coding—based on Lie algebra distilled to matrix multiplications—makes it easier to incorporate new algorithms. Currently, we are infusing this second phase with inertial disks to stabilize the ROV and artificial intelligence and control theory.

We will attempt this again. In today's world, international collaborations are as critical as new approaches to engineering. We have learned that a collaborative international bachelor project requires more intervention and guidance from the faculty, than a normal project. During the Utah IMECE conference, members from both teams will meet and discuss a more organized approach for next year's projects.

APPENDIX

$$B_{41} = \begin{pmatrix} R^{(1)}(t)\left(\overleftrightarrow{R^{(2/1)}(t)R^{(3/2)}(t)s_c^{(3)}}\right)^T + R^{(1)}(t)\left(\overleftrightarrow{R^{(2/1)}(t)s_j^{(2)}}\right)^T + \\ R^{(1)}(t)\left(\overleftrightarrow{R^{(2/1)}(t)s_c^{(2)}}\right)^T + R^{(1)}(t)\left(\overleftrightarrow{s_j^{(1)}}\right)^T \end{pmatrix} \tag{88}$$

$$B_{42} = \begin{pmatrix} R^{(1)}(t)R^{(2/1)}(t)\left(\overleftrightarrow{R^{(3/2)}(t)s_c^{(3)}}\right)^T + R^{(1)}(t)R^{(2/1)}(t)\left(\overleftrightarrow{s_j^{(2)}}\right)^T + \\ R^{(1)}(t)R^{(2/1)}(t)\left(\overleftrightarrow{s_c^{(2)}}\right)^T \end{pmatrix} e_{(2)} \tag{89}$$

$$B_{81} = \begin{pmatrix} R^{(1)}(t)\left(\overleftrightarrow{R^{(4/1)}(t)R^{(5/4)}(t)s_c^{(5)}}\right)^T + R^{(1)}(t)\left(\overleftrightarrow{R^{(4/1)}(t)s_j^{(4)}}\right)^T + \\ R^{(1)}(t)\left(\overleftrightarrow{R^{(4/1)}(t)s_c^{(4)}}\right)^T + R^{(1)}(t)\left(\overleftrightarrow{s_j^{(3)}}\right)^T \end{pmatrix} \tag{90}$$

$$B_{84} = R^{(1)}(t)R^{(4/1)}(t)\begin{pmatrix} \left(\overleftrightarrow{R^{(5/4)}(t)s_c^{(5)}}\right)^T + \\ \left(\overleftrightarrow{s_j^{(4)}}\right)^T + \left(\overleftrightarrow{s_c^{(4)}}\right)^T \end{pmatrix} e_{(2)} \tag{91}$$

$$B_{85} = R^{(1)}(t)R^{(4/1)}(t)R^{(5/4)}(t)\left(\overleftrightarrow{s_c^{(5)}}\right)^T e_1 \tag{92}$$

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